

## 7.2 Graph Coloring

### 7.2.1 Map Coloring<sup>3</sup>

<b>Description</b>	Students find the smallest number of colors needed to color a map and connect their reasoning to graph theory.
Core Math	Graph coloring
Standards for Mathematical Practices	MP 1: Make Sense and Persevere MP 3: Construct Viable Argument and Critique the Reasoning of Others MP 4: Model with Mathematics
Resources and Setup	Copies of Helping the Aussies worksheet Colored Pencils
Vocabulary	<ul style="list-style-type: none"> <li>• <b>Coloring</b> - A coloring of a graph is an assignment of colors to the vertices.</li> <li>• <b>Proper Coloring</b> - A coloring is proper if each vertex has a different color than the vertices adjacent to it.</li> <li>• <b>Chromatic number</b> - The chromatic number of a graph <math>G</math> is the minimum number of colors needed to properly color the vertices of <math>G</math>.</li> <li>• <b>Isolated vertex</b> - A vertex <math>v</math> is an isolated vertex if it has no neighbors.</li> <li>• <b>Connected</b> – A graph is connected if, for any two vertices <math>u</math> and <math>v</math> in the graph, there exists a walk between those two vertices.</li> <li>• <b>k-colorable</b> - a graph <math>G</math> is <math>k</math>-colorable if the chromatic number of <math>G</math> is less than or equal to <math>k</math></li> </ul>

<sup>3</sup>Source: Swinyard, Craig (2002). *An Introductory Graph Theory Curriculum* (Unpublished master's thesis). Portland State University, Portland, OR.

Lesson	
Launch	<p>Talk about the logistics of mapmaking and its coloring constraints- regions that share a border must be colored in different colors.</p> <p>Display an uncolored map of the USA  <a href="http://www.thecolor.com/Coloring/United-States-Map.aspx">http://www.thecolor.com/Coloring/United-States-Map.aspx</a> and discuss as a class:            What is the smallest number of colors that you think you would need to color a map of the USA?</p>
Instructional Activities	<p>Have students work on the Helping the Aussies task individually, then compare their results in groups.</p>
Share	<p>Select students to share their maps and reasoning for why their number of colors is minimal.</p>
Summarize	<p>Teacher shows how to represent the Australia map as a graph, with a vertex for each state and an edge between adjacent states.</p> <p>Introduce formal definitions of coloring, proper coloring, chromatic number, and isolated vertex, and give students time to add them to their graph theory graphic organizer.</p>

## Helping the Aussies

Australia is comprised of eight states and territories. To save money when making maps of Australia, a mapmaker wants to know the minimum number of colors needed to color the map in such a way that no two neighboring territories have the same color. (Two territories are said to neighbor each other if they share a border longer than just a single point.)

What is the least number of colors needed? \_\_\_\_\_



How do you know that you cannot color the map using fewer colors?

## 7.2.2 Roses are Red, Violets are Blue

<b>Description</b>	Students find the chromatic number of graphs and construct graphs with certain chromatic numbers
Core Math	Graph coloring
Standards for Mathematical Practices	MP 1: Make Sense and Persevere MP 3: Construct Viable Argument and Critique the Reasoning of Others MP 4: Model with Mathematics MP 7: Look for and Make Use of Structure
Resources and Setup	Copies of Roses are Red, Violets are Blue worksheet Colored Pencils Poster paper  Teacher Resources <ul style="list-style-type: none"><li>• Extension – Chromatic Number investigation</li></ul>
New Vocabulary	<ul style="list-style-type: none"><li>• <b>Planar</b> - A graph is said to be planar if it can be drawn in such a way that no two edges cross.</li></ul>

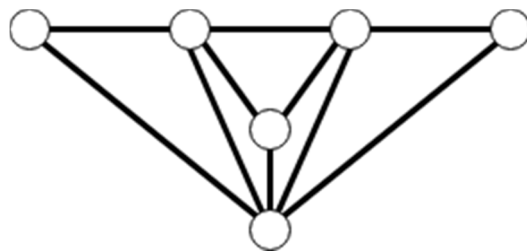
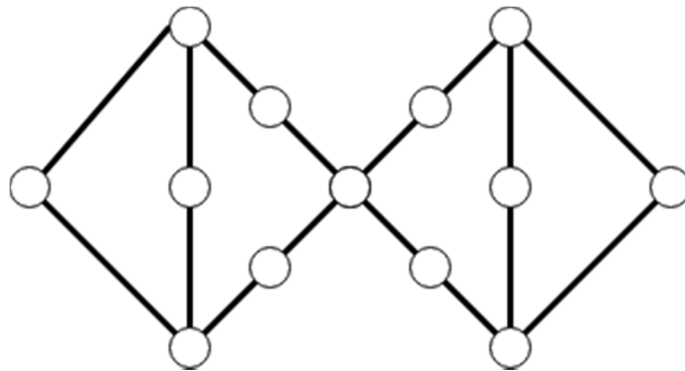
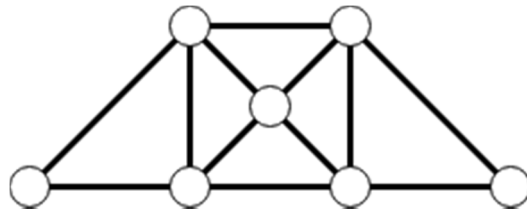
Lesson	
Launch	Discuss the logistics of the garden plans in terms of graph theory. What does each vertex represent? What does it mean (in terms of the garden) for two vertices to be adjacent?
Instructional Activities	Students work in groups on Roses are Red task, finding minimal colorings of each of the provided garden arrangements, then creating their own.
Share	<p>Students make posters of their designs and share out about the process of coming up with the design as well as difficulties along the way of the design process.</p> <p>After viewing the posters, have students reflect on what features of a graph made it require 2 colors, 3 colors, or 4 colors.</p>
Summarize	<p>Connect student observations to the existence of cliques. If a graph has a clique with <math>k</math> vertices, it requires at least <math>k</math> colors.</p> <p>Show video on the Four Color Theorem (<a href="https://www.youtube.com/watch?v=ANY7X-wpNs">https://www.youtube.com/watch?v=ANY7X-wpNs</a> )</p>
Extensions	<p>Two options for extending to an extra day:</p> <p>Students could return to the problem of coloring the USA map and find its chromatic number.</p> <p>Students could also investigate the chromatic number of various types of graphs (cycles, complete graphs, bipartite graphs) using the Chromatic Number investigation in the Teacher Resources folder</p>
Homework	Practice finding the chromatic number of various graphs
Notes	The vocabulary is essential to keep organized. Have students reference their graphic organizer.

## Roses are Red, Violets are Blue

Max the Landscaper has a new contract for a flower garden at the new city library.

The city planner is looking to keep costs down, so they asked Max to present to them a selection of designs for the flower garden with the minimum number of different flowers. The planner is precise in the requirements of the library flower garden. The city planner wanted a variety of colors however, no two colors could be next to each other.

Max came up with three possible patterns. What is the minimum number of colors could Max use in each design?



The city planner likes what she sees in Max's ideas, so she's asked Max to design a few more flower garden options but this time with more flower pots. Max needs your help to finish his proposal for the city planner. The city planner wants plans for 12 flower pots. Design a flower garden that requires 2 colors, one that requires 3 colors, and one that requires 4 colors. Can you design one that requires more than 4 colors?